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Polarization stability of TE and TM waves in nonlinear planar waveguides

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By means of linear analysis, we analytically show that in nonlinear planar waveguiding structures, TE_0 guided modes are stable to TM perturbations, and vice versa, TM_0 guided modes are stable to TE perturbations.

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I. INTRODUCTION

The propagation of TE waves in nonlinear planar waveguides has been the subject of extensive investigations in recent years [1–3]. Many device applications have been proposed based on the power-dependent properties of these waveguides. The existing rich literature on TE waves has been made possible mainly by the fact that TE waves have only one electric-field component whose governing equation is amenable to analytical solution. TM waves have not enjoyed the same degree of attention because of the presence of two electric-field components which often require extensive numerical computation, although several authors have recently addressed some aspects of stationary TM and coupled TE-TM propagation [2,4].

In linear waveguides, all guided waves are implicitly (and neutrally) stable. But in nonlinear waveguides, the nonlinearity brings in, along with many potentially useful features, an uncertainty, about whether a certain stationary state can propagate stably over a practically useful distance. This crucial question of stability has been addressed by a number of authors in recent years [1–12]. But as far as we know, these investigations have not included cases in which perturbations to a mode (or stationary wave) can be polarized in directions other than that of the mode. The aim of the present paper is to study the stability of a stationary wave in one type of polarization in the presence of perturbations in other polarizations, in waveguiding structures with planar geometry. This is probably the first study on nonstationary characteristics of guided waves having TM component. We have found that a nonlinear TE mode, polarized in the y direction, is not affected by small perturbations in the x- and z-directions. In other words, if it is stable (or unstable) to small perturbations in its own polarization, then it remains stable (or unstable) in the presence of perturbations in all polarizations. A similar result has been found for nonlinear planar TM guided waves.

We note that Shen, Stegeman, and Maradudin [5] have investigated the possibility of controlling a weak TM wave by a strong nonlinear TE wave but relied on the *a priori* assumption that the strong TE wave is totally unaffected by the weak TM wave. Boardman and Twardowski [6] later relaxed the constraint on the relative smallness of the TM component but again assumed that the coupled TE-TM wave propagates in a stationary manner. In addition to analytically *proving* that the assumption used in Ref. [5] is indeed valid, the result of our present work further indicates that it may also be possible to have the reverse situation, i.e., to control weak TE waves by strong nonlinear TM waves.

II. BASIC EQUATIONS

We are studying planar structures which may consist of several layers of Kerr-law materials. Guided waves propagate in the z direction and are uniform in the y direction, while the x axis is perpendicular to the layers.

In linear planar waveguide theory, if e_j denotes any of the e_x, e_y, e_z components of the electric field $\mathbf{E}$, then the governing wave equation in the weak-guidance approximation can be written simply as (see, e.g., [3])

$$Le_j = 0 \ , \tag{1}$$

where

$$L = 2i\beta\frac{\partial}{\partial z} + \frac{\partial^2}{\partial x^2} + (n_L^2 k^2 - \beta^2) \ , \tag{2}$$

n_L is the linear refractive index, k is the free-space wave number, and β is the propagation constant.

When one or more of the layers are nonlinear, the full-vector wave equation has a general form

$$\nabla\times\nabla\times\mathbf{E} - k^2 n_L^2 \mathbf{E} = \frac{k^2}{\epsilon_0}\mathbf{P}_{NL} \ , \tag{3}$$

where $\mathbf{E}$ is the electric-field vector, ϵ_0 is the free-space permittivity, and $\mathbf{P}_{NL}$ is the nonlinear polarization vector. When $\mathbf{P}_{NL}$ vanishes (as in a linear medium), Eq. (3) reduces to (1) in the weak-guidance limit. It is well known (see, e.g., [13]) that in a Kerr-law medium, the (third-order) polarization $\mathbf{P}_{NL}$ is related to $\mathbf{E}$ through

$$\mathbf{P}_{NL} = A[(\mathbf{E}\cdot\mathbf{E}^*)\mathbf{E} + \eta(\mathbf{E}\cdot\mathbf{E})\mathbf{E}^*] \ , \tag{4}$$

where $A = 6\chi_{1122}$ in the notation of Ref. [14], and $\eta = 3$, $\frac{1}{2}$, or 0 for nonlinear mechanisms arising from molecular orientation, nonresonant electronic response, or electrostriction, respectively. In this paper, we are concerned only with the case $\eta = \frac{1}{2}$.

If $\mathbf{E}$ has only one component (such as for TE waves), then $\mathbf{P}_{\rm NL} \propto |\mathbf{E}|^2\mathbf{E}$, and Eq. (3) takes a simpler form

$$\nabla\times\nabla\times\mathbf{E}-k^2n_{\rm NL}^2\mathbf{E}=\mathbf{0} , \tag{5}$$

with the Kerr refractive index $n_{\rm NL}$ being given by

$$n_{\rm NL}^2=n_{\rm L}^2+\alpha|\mathbf{E}|^2 ,$$

and α is a nonlinear coefficient. If $\mathbf{E}$ has all three components e_x, e_y, and e_z in the x, y, and z directions, respectively, then the expansion for $\mathbf{P}_{\rm NL}$ is

$$\mathbf{P}_{\rm NL}=A[P_{\rm NL}^x\hat{\mathbf{x}}+P_{\rm NL}^y\hat{\mathbf{y}}+P_{\rm NL}^z\hat{\mathbf{z}}] , \tag{6}$$

where

$$P_x^{\rm NL}=\tfrac{3}{2}e_x|e_x|^2+e_x(|e_y|^2+|e_z|^2)+\tfrac{1}{2}e_x^*(e_y^2+e_z^2) , \tag{7}$$

$$P_y^{\rm NL}=\tfrac{3}{2}e_y|e_y|^2+e_y(|e_x|^2+|e_z|^2)+\tfrac{1}{2}e_y^*(e_x^2+e_z^2) , \tag{8}$$

$$P_z^{\rm NL}=\tfrac{3}{2}e_z|e_z|^2+e_z(|e_y|^2+|e_x|^2)+\tfrac{1}{2}e_z^*(e_y^2+e_x^2) . \tag{9}$$

We then assume that the first term of Eq. (5) can still be replaced by $-\nabla^2\mathbf{E}$, in the spirit of the weak-guidance approximation. This is justified when it is recalled that the scalar approximation is well-known to be a very good approximation in most real waveguides, and that the nonlinear change of the refractive index in real materials is often much less than 0.01 [15]. In the particular case of TE waves, this replacement is exact without recourse to the weak-guidance approximation.

III. TE GUIDED WAVES

We now consider a TE mode which is y polarized with field $E_0(x)\exp i(\beta z-\omega t)$. Let e_y be expressed as

$$e_y=E_0(x)+\delta e_y(x,z) , \tag{10}$$

where δe_y is a small perturbation in the y polarization. We also assume that e_x, e_z are small perturbations in the x and z polarizations. In linear stability analysis, the following question is to be answered: with $E_0(x)$ fixed and e_x, δe_y, e_z initially confined to small values, do these perturbations grow with propagation distance? If they do then E_0 is regarded as unstable. Otherwise, it is stable.

Now, from the above equations and assumptions, the equations governing the small perturbations, to first order, can be written as

$$L\delta e_y+\alpha k^2E_0^2(2\delta e_y+\delta e_y^*)=0 , \tag{11}$$

$$Le_j+\tfrac{1}{3}\alpha k^2E_0^2(2e_j+e_j^*)=0 , \tag{12}$$

where, here, e_j denotes either e_x or e_z. In particular, Eq. (11) does not involve e_x and e_z, and has been studied extensively in the context of purely TE perturbations [9–12]. It has been shown that for TE_0 modes, growth rates of TE perturbations in linear analysis can only be real (i.e., the mode is unstable) or purely imaginary (stable); while for higher-order TE modes, growth rates can be complex [12], indicating a complicated structure of stability regions in the parameter space. It should also be mentioned that, for TE_0 modes, the determination of stability can be facilitated by a simple topological rule provided that the stability at some particular set of parameters is known [16].

Of primary interest in the present study is how Eq. (12), which differs from Eq. (11) only in the appearance of the factor $\frac{1}{3}$ in the last term, dictates the behavior of e_x and e_z. It turns out that this simple factor plays a crucial role in the stability analysis.

Following Ref. [9], e_j can be the form

$$e_j(x,z)=(u+v)\exp(\mu z)+(u^*-v^*)\exp(\mu^*z) ,$$

where u,v are functions of x only, and μ represents the growth rate. Some straightforward algebra leads to familiar equations

$$L_0v=-i\Omega u,\quad L_1u=-i\Omega v , \tag{13}$$

$$L_0L_1u=-\Omega^2u,\quad L_1L_0v=-\Omega^2v , \tag{14}$$

where

$$L_0=d^2/dx^2+(n_L^2k^2-\beta^2)+\tfrac{1}{3}\alpha k^2E_0^2 ,$$

$$L_1=L_0+\tfrac{2}{3}\alpha k^2E_0^2 , \tag{15}$$

and $\Omega=2\mu\beta$. These forms of L_0 and L_1 differ from those of the purely TE case [8–12] in the appearance of the $\frac{1}{3}$ term, with the following effect: the present L_0 and u, respectively, play the role of L_1 and v in Refs. [8–12], and likewise, here, L_1 and v play the role of L_0 and u in [8–12]. The crucial difference is that L_1 as defined by (15) has a zero eigenvalue with eigenfunction E_0 and all other eigenvalues are negative, *but L_0 is negative definite.* While details are exiled to the Appendix, it can be shown that, accordingly, Ω (and therefore the growth rate μ) is purely imaginary, which implies that E_0 is stable to perturbations e_x and e_z.

From a physical perspective, the $\frac{1}{3}$ factor means that the nonlinearity acting upon the perturbations e_x and e_z is only one third of that acting upon δe_y (which is a perturbation in the same polarization as the E_0 mode) and hence not large enough to influence stability. Thus small perturbations in the x and z polarization do not grow with propagation distances, indicating that weak TM waves can be guided by strong nonlinear TE modes.

IV. TM GUIDED WAVES

Stationary TM waves have $e_y=0$, and the dominant component is e_x. In analogy to $E_0(x)$, we let $E_0^x(x)$ and $E_0^z(x)$ (which are $\pi/2$ out of phase) denote the two components of a TM mode, δe_x, δe_z, respectively, be perturbations to these components, and e_y is regarded as a small perturbation in the y polarization.

For δe_x, δe_z, the governing equations are

$$L\delta e_x+\alpha k^2|E_0^x|^2(2\delta e_x+\delta e_x^*)+\tfrac{1}{3}\alpha k^2|E_0^z|^2(2\delta e_x-\delta e_x^*)=0 , \tag{16}$$

$$L\delta e_z+\alpha k^2|E_0^z|^2(2\delta e_z+\delta e_z^*)+\tfrac{1}{3}\alpha k^2|E_0^x|^2(2\delta e_z-\delta e_z^*)=0 , \tag{17}$$

which do not involve e_y; while for e_y,

$$Le_y+\tfrac{1}{3}\alpha k^2E_0^2(2e_y+e_y^*)=0\ . \qquad (18)$$

It is often the case that E_0^x is larger than E_0^z by an order of magnitude or more [4] so that the last term of Eq. (16) and the second term of (17) can be ignored. This would then lead to the well-studied Eq. (11) or Eq. (12) discussed above. Even if E_0^z can not be ignored, the important fact is that Eqs. (16) and (17) do not involve e_y (and we make no attempt to study them here), and hence polarization stability is determined entirely by Eq. (18). Since Eq. (18) is identical to (12), the same conclusion is reached: TE perturbations do not affect the stability of TM guided modes.

It should be mentioned that this conclusion is for TE_0 and TM_0 modes only, because for higher-order modes L_1 can have positive as well as negative eigenvalues, indicating that growth rates can be complex [12].

V. CONCLUSION

We have established that, in nonlinear planar optical guiding structures, TE guided waves are stable to TM perturbations, and vice versa, TM guided waves are stable to TE perturbations. These results indicate that in practice, it may be possible to excite and propagate TE and TM guided waves over long distances in nonlinear planar waveguides. The fact that small perturbations in directions other than that of a nonlinear mode do not grow is consistent with a recent work by Boardman *et al.* in Ref. [6] which shows that stationary coupled TE-TM waves are possible in nonlinear planar structures.

APPENDIX

The following derivation is similar to that of Kolokolov in Refs. [8] and [11] with a few minor modifications, and is given here only for the sake of convenience of reference.

Let $\langle g_1,g_2\rangle$ be defined as

$$\langle g_1,g_2\rangle=\int_{-\infty}^{\infty}g_1^*g_2dx$$

for continuous functions g_1, g_2 which decay at $x=\pm\infty$. Since $L_1L_0v=-\Omega^2v$ and $L_1E_0=0$ (as E_0 is a nonlinear mode), we have

$$-\Omega^2\langle v,E_0\rangle=\langle L_1L_0v,E_0\rangle=\langle v,L_0L_1E_0\rangle=0\ ,$$

i.e., v is orthogonal to E_0. Also, one can write

$$-\Omega^2=\frac{\langle v,L_0v\rangle}{\langle v,L_1^{-1}v\rangle}=\lambda\ (\text{say})\ .$$

In the function space orthogonal to E_0, $\langle v,L_1^{-1}v\rangle$ is negative definite, and the stability of E_0 depends on the sign of the maximum value of $\langle v,L_0v\rangle$. If this sign is negative, then Ω is purely imaginary and E_0 is stable. Otherwise, it is unstable.

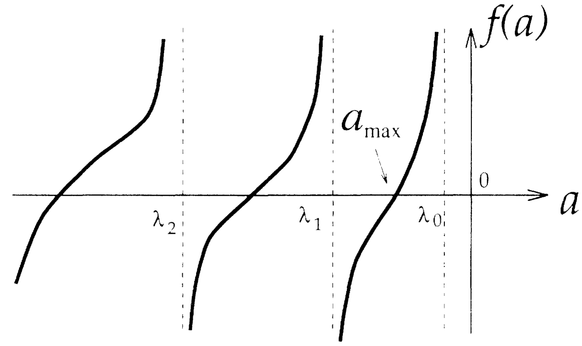

FIG. 1. Schematic behavior of $f(a)=\sum_i|c_i|^2/(\lambda_i-a)$.

To determine the maximum of $\langle v,L_0v\rangle$, the method of Lagrangian multipliers can be employed in which a functional

$$f=\langle v,L_0v\rangle-a\langle v,v\rangle-b(\langle v,E_0\rangle+\langle E_0,v\rangle)$$

is formed, where a and b are the undetermined multipliers. Taking the first variation of f with respect to v gives

$$L_0v=av+bE_0\ , \qquad (A1)$$

from which $a=\langle v,L_0v\rangle$ whose maximum is what we require. Let

$$v=\sum_i\alpha_i\omega_i,\quad E_0=\sum_i c_i\omega_i\ ,$$

where ω_i are the eigenfunctions (with corresponding eigenvalues λ_i) of L_0, (A1) becomes

$$\sum_i\alpha_i\lambda_i\omega_i=a\sum_i\alpha_i\omega_i+b\sum_i c_i\omega_i$$

and, therefore,

$$\alpha_i=\frac{bc_i}{\lambda_i-a},\quad v=\sum_i\frac{bc_i}{\lambda_i-a}\omega_i\ .$$

Thus,

$$\langle E_0,v\rangle=0=\sum_i\frac{b|c_i|^2}{\lambda_i-a}=bf(a)\ (\text{say})\ .$$

The schematic behavior of $f(a)$ is shown in Fig. 1 in which λ_0 and λ_1 are the first two singularities. It is obvious that the maximum value of a, denoted by a_{max}, satisfies $\lambda_1<a_{max}<\lambda_0$. But since $L_0=L_1-\frac{2}{3}n_2k^2E_0^2$ $(n_2>0)$ and L_1 has no positive eigenvalues, all eigenvalues of L_0, including λ_0 and λ_1, must be negative. Hence the maximum value of $a=\langle v,L_0v\rangle$ satisfying $f(a)=0$ must also be negative.

[1] D. Mihalache, M. Bertolotti, and C. Sibilia, in *Progress in Optics,* edited by E. Wolf (North-Holland, Amsterdam, 1989), p. 229, and references therein.
[2] A. D. Boardman, P. Egan, F. Lederer, U. Langbein, and D. Mihalache, in *Nonlinear Surface Electromagnetic Phenomena,* edited by H.-E. Ponath and G. I. Stegeman (North-Holland, Amsterdam, 1991), Chap. 2, and references therein.
[3] A. C. Newell and J. M. Moloney, *Nonlinear Optics* (Addison-Wesley, Reading, 1992).
[4] K. Ogusu, IEEE Trans. Microwave Theory Technol. **37**, 941 (1989).
[5] T. P. Shen, G. I. Stegeman, and A. A. Maradudin, Appl. Phys. Lett. **52**, 1 (1988).
[6] A. D. Boardman and T. Twardowski, Phys. Rev. A **39**, 2481 (1989).
[7] C. K. R. T. Jones and J. V. Moloney, Phys. Lett. A **117**, 175 (1986).
[8] A. A. Kolokolov, Lett. Nuovo Cimento **8**, 197 (1973).
[9] N. V. Vysotina, N. N. Rozanov, and V. A. Smirnov, Sov. Phys. Tech. Phys. **32**, 104 (1987).
[10] N. N. Akhmediev and N. V. Ostrovskaya, Sov. Phys. Tech. Phys. **33**, 1333 (1988).
[11] H. T. Tran, Opt. Commun. **93**, 202 (1992).
[12] H. T. Tran, J. D. Mitchell, N. N. Ahkmediev, and A. Ankiewicz, Opt. Commun. **93**, 227 (1992).
[13] R. W. Boyd, *Nonlinear Optics* (Academic, New York, 1992).
[14] P. D. Maker and R. Terhune, Phys. Rev. **137**, A801 (1964).
[15] G. I. Stegeman, E. M. Wright, N. Finlayson, R. Zanoni, and C. T. Seaton, J. Lightwave Technol. **6**, 953 (1988).
[16] J. D. Mitchell and A. W. Snyder, J. Opt. Soc. Am. B **10**, 1572 (1993).